Simulating Multiwinner Voting Rules in Judgment Aggregation: (Extended Abstract)*

Julian Chingoma, **Ulle Endriss**, **Ronald de Haan**
ILLC, University of Amsterdam
{j.z.chingoma, u.endriss, r.dehaan}@uva.nl

1 Introduction

Various formal frameworks have been proposed to study collective decision making, where agents report an individual view and a single collective view that presents a reasonable compromise must be determined. To better understand these mechanisms, each proposed for different settings, and to enable the transfer of knowledge between domains of collective decision making, it is important to isolate fundamental building blocks that are common to different solutions.

In this paper, we investigate the extent to which it is possible to simulate *multiwinner voting rules*, i.e., voting rules to elect a committee of representatives (Faliszewski et al. 2017), within the framework of logic-based *judgment aggregation* (JA) (List and Pettit 2002; Grossi and Pigozzi 2014; Endriss 2016), and specifically, the JA model with *rationality* and *feasibility constraints* (Endriss 2018). Embedding a multiwinner voting rule into the expressive JA framework makes it very natural to meticulously study refinements of standard rules. To address the added computational demands associated with JA, we make use of results centred on Boolean circuits in *decomposable negation normal form* (de Haan 2018). Also, we take steps in importing the well-studied multiwinner voting concept of *proportionality* into JA, but this work is omitted from this extended abstract as we highlight the work that is most relevant to the Knowledge Representation and Reasoning community.

2 Preliminaries

The multiwinner voting model concerns a finite set of *alternatives* X and a set of *agents* $N = \{1, \dots, n\}$. For the sake of simplicity, we shall assume n is odd. Each agent $i \in N$ has a weak preference order $\succcurlyeq_i$ on X, with $\succ_i$ denoting the strict part of $\succcurlyeq_i$. With weak preferences, we can consider two preference types a voter may provide: (strict) *ordinal* (a strict ranking of alternatives) and *approval-based* (a set of alternatives she approves of). A *voting rule* maps a *profile* of such preferences, one for each agent, to a set of winning committees (i.e., ties are possible).

Let us detail some multiwinner voting rules. The k-*Borda* rule elects the committee/s of the k alternatives with the highest Borda scores, defined as $B(x) = \sum_{i \in N} |\{y \in X \mid x \succ_i y\}|$ for alternative x. The k-*Copeland* rule selects the committee/s that consist of the k alternatives with the highest Copeland scores, defined as $C(x) = |\{y \in X \mid x \succ_M y\}| - |\{y \in X \mid y \succ_M x\}|$, where $\succ_M$ is the strict majority relation ($x \succ_M y$ if and only if $|\{i \in N \mid x \succ_i y\}| > n/2$). Next, we mention one of the approval-based rules that we study, which belongs to the family of *Thiele rules* (Thiele 1895): the (simple) *Approval Voting* (AV) rule outputs the k most approved alternatives. Finally, we mention *Gehrlein-stable* rules. A committee A is (weakly) Gehrlein-stable if, for any $x \in A$ and $y \in X \setminus A$, it is the case that $|\{i \in N \mid x \succ_i y\}| \geqslant |\{i \in N \mid y \succ_i x\}|$. Gehrlein-stable rules return such committees whenever they exist.

Next, we present the JA model. A *judgment* is a function that expresses the acceptance or rejection of each of the issues in the *agenda* Φ, a finite set of propositional atoms. Each agent submits such a judgment. An *aggregation rule* F maps any given vector of the agents' judgments, called the JA *profile*, to a nonempty set of judgments. We use formulas from the propositional language $\mathcal{L}(\Phi)$ to express constraints on judgments, including both *rationality* constraints, i.e., constraints on acceptable inputs to F, and *feasibility* constraints, i.e., constraints indicating what are acceptable outputs of F.

We consider two well-known JA rules. The *max-num rule* (also known as the Slater rule) selects judgments for which the number of agreements with the majority outcome is maximal, while the *max-sum rule* (also known as the Kemeny rule) maximises the sum of the agreements with the profile. Both rules are part of the class of *additive majority rules* (AMRs) which, loosely speaking, return outcomes in favour of the majority to various degrees.

3 Simulation of Multiwinner Rules

Given a set of alternatives X, we let $\Phi^X_{\succcurlyeq} = \{p_{x \succcurlyeq y} \mid x, y \in X\}$ be the *preference agenda* (Dietrich and List 2007; Endriss 2016). We can think of accepting the proposition $p_{x \succcurlyeq y}$ as expressing a (weak) preference of x over y. Furthermore, we can use $p_{x \succ y}$ as a shorthand for $p_{x \succcurlyeq y} \wedge \neg p_{y \succcurlyeq x}$.

We can then express properties of binary relations as constraints in our logical language, each defined for some set $A \subseteq X$. This includes common properties of preference relations such as completeness, antisymmetry, and transitivity:

*This is an extended abstract of a paper originally presented at the 21st International Conference on Autonomous Agents and Multiagent Systems (Chingoma, Endriss, and de Haan 2022).

Multiwinner Rule	Rationality Constraint	Feasibility Constraint	JA Rule
k-Borda	RANKING	k-INDIFF-INDIFF / k-INDIFF-INCOMP	max-sum
k-Copeland	RANKING	k-INDIFF-INDIFF / k-INDIFF-INCOMP	max-num
AV	INDIFF-INCOMP	k-INDIFF-INCOMP	max-sum
Gehrlein-stable rules	RANKING	k-INCOMP-INCOMP	AMR

Table 1: A summary of our results on the simulation of multiwinner voting rules in JA. For each multiwinner voting rule, we specify which JA rule in combination with which pair of rationality and feasibility constraints enables its simulation.

$$\text{COMPLETE}_A = \bigwedge_{x,y \in A} (p_{x \succcurlyeq y} \vee p_{y \succcurlyeq x})$$

$$\text{ANTI-SYM}_A = \bigwedge_{x,y \in A \text{ s.t. } x \neq y} \neg(p_{x \succcurlyeq y} \wedge p_{y \succcurlyeq x})$$

$$\text{TRANSITIVE}_A = \bigwedge_{x,y,z \in A} (p_{x \succcurlyeq y} \wedge p_{y \succcurlyeq z} \rightarrow p_{x \succcurlyeq z})$$

We can formulate a constraint that is satisfied by a judgment that corresponds to a strict ranking of all alternatives:

$$\text{RANKING} = \text{COMPLETE}_X \wedge \text{ANTI-SYM}_X \wedge \text{TRANSITIVE}_X$$

For two alternatives, instead of having a strict preference for one over the other, one may also be *indifferent* between them. Or, one may consider the alternatives to be *incomparable*. The following constraints describe these notions:

$$\text{INDIFF}_A = \bigwedge_{x,y \in A} (p_{x \succcurlyeq y} \wedge p_{y \succcurlyeq x})$$

$$\text{INCOMP}_A = \bigwedge_{x,y \in A} \neg(p_{x \succcurlyeq y} \vee p_{y \succcurlyeq x})$$

And the following constraint expresses a strict preference for all alternatives within some $A \subseteq X$ over all those not in A:

$$\text{TOP}_A = \bigwedge_{x \in A} \bigwedge_{y \in X \setminus A} (p_{x \succ y})$$

We are going to require constraints to describe both that (i) there exists a set A of most preferred alternatives and that (ii) there exists such a set and that this set has size k. In both cases, we may assume either indifference between all the alternatives within the same set, or all of these alternatives being incomparable. We can define a constraint that requires indifference in the top set and incomparability in the bottom set as follows:

$$\text{INDIFF-INCOMP} = \bigvee_{A \in \mathcal{P}_+(X)} (\text{TOP}_A \wedge \text{INDIFF}_A \wedge \text{INCOMP}_{X \setminus A})$$

Here $\mathcal{P}_+(X)$ denotes the set of all nonempty subsets of X. The constraint INDIFF-INDIFF and INCOMP-INCOMP can be defined similarly, with further constraints, such as k-INDIFF-INDIFF, k-INDIFF-INCOMP and k-INCOMP-INCOMP, being additionally prefixed with the number k to indicate that the top set has size k.

To simulate a multiwinner voting rule, the agents' preferences are turned into judgments that satisfy a suitable rationality constraint (RANKING for ordinal ballots, and INDIFF-INCOMP for approval ballots). We can then apply a JA rule to the preferences thus encoded that satisfies a feasibility constraint that ensures that all of its outcomes satisfy TOP_A for some $A \subseteq X$ (we can declare the alternatives in A the winners of the original election). Our main simulation results are summarised in Table 1.

4 Constraints as Circuits

Worst-case intractability, specifically Θ_2^p-hardness, has been shown for max-sum and max-num (Endriss et al. 2020). Thus, to maintain the efficiency of some rules we simulate, such as k-Borda and AV (Aziz et al. 2015; Elkind et al. 2017), we leverage results that hinge on circuits in *decomposable negation normal form* (DNNF circuits) (de Haan 2018). A DNNF circuit is a Boolean circuit such that $\top$, $\bot$, x or $\neg x$ for a propositional variable x are leaf-node labels ($\wedge$ and $\vee$ are internal nodes), and it satisfies decomposability: for each conjunction, no two conjuncts share a propositional variable (Darwiche and Marquis 2002).

The result of de Haan (2018) tells us that, if the integrity constraint is represented as a DNNF circuit, then computing the outcomes for both max-sum and max-num is polynomial-time solvable. For the two main constraints used for our simulation results, we obtain that we indeed can encode k-INDIFF-INCOMP into a DNNF circuit in polynomial time. Unsurprisingly, the same holds for INDIFF-INCOMP. These results ensure that for max-sum and max-num, when using k-INDIFF-INCOMP as the feasibility constraint, computing the outcomes can still be done in polynomial time. However, we find that computing any outcome for the max-sum rule induced by k-INDIFF-INDIFF, on $\top$-restricted ballots, is NP-hard. This rules out efficient use of such a rule in general. But, when used on RANKING-restricted profiles, it simulates k-Borda so in this particular case, we know that the rule obtained nevertheless is computationally efficient. This highlights the care required in JA constraint selection.

5 Future Work

In addition to the work outlined in this extended abstract, the full paper not only details our JA simulation of some well-known, approval-based multiwinner rules for proportionality, but also presents a proposal for transferring the notion of proportionality, which is well-studied in multiwinner voting, to the domain of JA, where it has not been considered before (Chingoma, Endriss, and de Haan 2022).

Our investigation then suggests multiple research paths. Moving forward, the JA simulation of more sophisticated multiwinner voting rules, such as sequential rules, should be explored. For the newly-proposed JA rules, the development of approximate versions of the rules can be pursued. Also, JA rules could aid the enrichment of multiwinner voting with new rules satisfying notions other than committee stability.

References

Aziz, H.; Gaspers, S.; Gudmundsson, J.; Mackenzie, S.; Mattei, N.; and Walsh, T. 2015. Computational aspects of multi-winner approval voting. In *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2015)*, 107–115.

Chingoma, J.; Endriss, U.; and de Haan, R. 2022. Simulating multiwinner voting rules in judgment aggregation. In *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2022)*.

Darwiche, A., and Marquis, P. 2002. A knowledge compilation map. *Journal of Artificial Intelligence Research* 17:229—-264.

Dietrich, F., and List, C. 2007. Arrow's theorem in judgement aggregation. *Social Choice and Welfare* 29(1):19–33.

Elkind, E.; Faliszewski, P.; Skowron, P.; and Slinko, A. 2017. Properties of multiwinner voting rules. *Social Choice and Welfare* 48(3):599–632.

Endriss, U.; de Haan, R.; Lang, J.; and Slavkovik, M. 2020. The complexity landscape of outcome determination in judgment aggregation. *Journal of Artificial Intelligence Research* 69:687–731.

Endriss, U. 2016. Judgment aggregation. In Brandt, F.; Conitzer, V.; Endriss, U.; Lang, J.; and Procaccia, A. D., eds., *Handbook of Computational Social Choice*. Cambridge University Press. chapter 17.

Endriss, U. 2018. Judgment aggregation with rationality and feasibility constraints. In *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2018)*, 946–954.

Faliszewski, P.; Skowron, P.; Slinko, A.; and Talmon, N. 2017. Multiwinner voting: A new challenge for social choice theory. In Endriss, U., ed., *Trends in Computational Social Choice*. AI Access. chapter 2, 27–47.

Grossi, D., and Pigozzi, G. 2014. *Judgment Aggregation: A Primer*. Synthesis Lectures on Artificial Intelligence and Machine Learning. Morgan & Claypool Publishers.

de Haan, R. 2018. Hunting for tractable languages for judgment aggregation. In *Proceedings of the 16th International Conference on Principles of Knowledge Representation and Reasoning (KR-2018)*. AAAI Press.

List, C., and Pettit, P. 2002. Aggregating sets of judgments: An impossibility result. *Economics and Philosophy* 18(1):89–110.

Thiele, T. N. 1895. Om flerfoldsvalg. In *Oversigt over det Kongelige Danske Videnskabernes Selskabs Forhandlinger*, 414–441.